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### The temperature distribution in an internal combustion engine piston

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Monterey, California. Naval Postgraduate School

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THE TEMPERATURE DISTRIBUTION IN AN  
INTERNAL COMBUSTION ENGINE PISTON

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D. H. PALACIOS

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THE TEMPERATURE DISTRIBUTION IN AN INTERNAL  
COMBUSTION ENGINE PISTON

-

D. H. PALACIOS

THE TEMPERATURE DILUTION IS AN INTERFERING  
COMBUSTION ENGINE SYSTEM

D. E. BAGGIO

17

THE TEMPERATURE DISTRIBUTION IN AN INTERNAL  
COMBUSTION ENGINE PISTON

by

Daniel Hax Palacios  
Lieutenant, Chilean Navy

Submitted in partial fulfillment  
of the requirements  
for the degree of  
MASTER OF SCIENCE  
in  
MECHANICAL ENGINEERING

United States Naval Postgraduate School  
Monterey, California  
1952

514

THE TEMPERATURE DISTRIBUTION IN AN INTERNAL  
COMPRESSION ENGINE PISTON

p2

Temperature, Celsius 162  
Density, kg/m<sup>3</sup> 1.025  
Pressure, bar 1.025

Stabilized in steady state  
of the measurement  
for the degree of  
MASTER OF SCIENCE  
in  
MECHANICAL ENGINEERING

University of Massachusetts Lowell  
Monteith, California  
1925

This work is accepted as fulfilling  
the thesis requirements for the degree of  
MASTER OF SCIENCE  
in  
MECHANICAL ENGINEERING

from the  
United States Naval Postgraduate School

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P. J. Kiefer  
Chairman  
Department of Mechanical Engineering

Approved:

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R. S. Glasgow  
Academic Dean

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## PREFACE

This investigation was prepared during the period February - June 1952 at the United States Naval Postgraduate School, Monterey, California.

The subject was suggested to the author by previous experience in maintenance of small high-speed diesel engines, during which burnt pistons had to be replaced on several occasions.

The author wishes to extend his appreciation to Assistant Professor E. E. Drucker for his interest and advice throughout the development of the work.

REF ID: A11111111

This transcription was dictated by the following  
person - June 1955 at the United Nations Mass Population  
Survey, Monterrey, California.

The subject was described as the author of a previous  
experience in manufacture of small high-speed drives  
engines, during which time he had been engaged  
on several occasions.

The author uses his application to  
manufacture processor E. D. Director for his improved and  
advantageous placement of the work.

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Figure 1.

Figure 2.

Figure 3.

TABLE OF SYMBOLS

Symbol	Quantity	Units
$\alpha$	Thermal Diffusivity.	$\text{ft}^2 \text{ hr}^{-1}$
$\theta$	Temperature of the Disk.	F
$\theta_0$	Initial Temperature of Disk.	F
$\phi$	Temperature of Barrel.	F
$\phi_0$	Initial Temperature of Barrel.	F
$\rho$	Mass Density of Piston Material.	$\text{lb}_m \text{ ft}^{-3}$
$\tau$	Time	hr
b	Thickness of Barrel Wall.	ft
$c_p$	Specific Heat at Constant Pressure of Piston Material	$\text{B lb}_m^{-1} \text{ F}^{-1}$
F	Degrees Fahrenheit	F
$h_1$	Heat-transfer Coefficient from Gas to Upper Surface of Disk	$\text{B hr}^{-1} \text{ ft}^{-2} \text{ F}^{-1}$
$h_3$	Heat-transfer Coefficient from outside surface of barrel into media beyond the surface	$\text{B hr}^{-1} \text{ ft}^{-2} \text{ F}^{-1}$
i	Square Root of Minus One	
$J_0$	Bessel function of the First Kind of Order Zero	
$J_1$	Bessel function of the First Kind of Order One.	
k	Thermal Conductivity of Piston Material	$\text{B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1}$
l	Thickness of Disk	ft

## CHURCHES IN CHINA

$L_o$	Length of Barrel.	ft
$n$	Frequency of Gas Temperature Fluctuation	$hr^{-1}$
$r$	Radius	ft
$R_o$	Outside Radius of Disk	ft
$R_i$	Inside radius of Disk	ft
$t$	Temperature, Standard Scale	F
$t_o$	Initial temperature of Piston	F
$T_a$	Temperature of Cooling Medium Surrounding Barrel.	F
$T_{gas}$	Gas Temperature	F
$T_m$	Mean Temperature of Gases of Combustion.	F
$T_o$	Amplitude of Gas Temperature Fluctuation.	F
$Y_0$	Bessel Function of the second Kind of Order Zero.	
$Y_1$	Bessel Function of the second Kind of Order One.	
$z$	Length Measured from Tom of Disk Downwards.	ft

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## SUMMARY

The objective of this work was to establish by analytical means the temperature distribution in an internal combustion engine piston. For this purpose the piston was divided into a disk and a barrel, for which boundary conditions were expressed for the exterior surfaces and a common conical surface. The problem was solved in two parts: first, a solution was found for the case where the gas temperature was assumed representable by an average temperature; this solution was then modified to account for a sinusoidal fluctuation of gas temperature.

The resulting temperature distribution, within the limits of the approximations made, is:  
for the disk:

$$t = T_m + T_0 e^{-\beta \sqrt{\frac{\pi n}{\alpha}} \tau} \sin(2\pi n \tau - \beta \sqrt{\frac{\pi n}{\alpha}}) +$$

$$+ \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \theta_0 e^{-\alpha(f_p^2 + b_m^2)\tau} \left[ \cos b_m \beta + \frac{h_1}{k b_m} \sin b_m \beta \right] J_0(f_p \tau)$$

for the barrel:

$$t = T_a + T_0 e^{-\beta \sqrt{\frac{\pi n}{\alpha}} \tau} \sin(2\pi n \tau - \beta \sqrt{\frac{\pi n}{\alpha}}) +$$

$$+ \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} P_p e^{-\alpha(f_p^2 + b_m^2)\tau} \cos b_m (\beta - L_0) \left[ J_0(f_p \tau) + F_p Y_0(f_p \tau) \right]$$

The objective of this work is to apply the  
numerical methods to the numerical differentiation to an  
interpolation combination scheme. For this purpose the  
bisection was divided into a part, for which  
numerical configurations were adopted for the extraction  
satisfies and a common boundary condition was solved  
in two parts: first, a solution was found for the case where  
the gas temperature was assumed to be constant to account  
for a sinusoidal fluctuation of the temperature,  
The mean fluid temperature distribution, which is  
limits of the suboxications range, is  
for the disk:

$$+ \left( \frac{\pi^2}{4} \right) \{ - \tau x \pi \} \sin \frac{\pi x}{2} + b \cdot e^{-s \cdot \tau} + \frac{1}{\pi} = 0$$

$$(s_1) \bar{u} \left[ \frac{1}{\pi^2} \sin \frac{\pi x}{2} + \{ - \tau x \pi \} \right] \pi \left( \frac{1}{\pi^2} + \frac{1}{4} \right) \tau - \frac{1}{\pi} = 0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} +$$

for the part:

$$+ \left( \frac{\pi^2}{4} \right) \{ - \tau x \pi \} \sin \frac{\pi x}{2} e^{-s \cdot \tau} + b \cdot \tau = 0$$

$$(s_2) \bar{u} \left[ \frac{1}{\pi^2} \sin \frac{\pi x}{2} + \{ - \tau x \pi \} \right] \pi \left( \frac{1}{\pi^2} + \frac{1}{4} \right) \tau - \frac{1}{\pi} = 0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} +$$

## CHAPTER I

### INTRODUCTION

Upon analyzing a piston of an internal combustion engine with a viewpoint to understanding its behaviour as a heat dissipating member of the engine, it is readily realized that the piston absorbs a certain amount of heat through the upper surface of its crown. This heat is partly stored in the body of the piston until it reaches its steady state temperature; at the same time there is heat dissipation from the outside surface of the barrel and rings and also through its inside surfaces. After the steady state is reached, all heat absorbed must be dissipated through these surfaces.

According to O. L. Adams (1), five percent of the heat liberated by the combustion of the fuel must pass through the piston crown surface into the piston. Of this heat, ten percent is dissipated by the lower surface of the crown of the piston, as stated by J. L. Hepworth (5).

Radiation from the gases into the piston only takes place through a very small portion of the time cycle and it amounts, according to B. Pinkel (9), to only ten percent of the total heat being absorbed by all the metallic parts forming the container for the hot gases; thus it may be seen that the heat absorbed by the piston itself through radiation will be a part of this ten percent depending on the relative areas of the piston crown with respect to the area of the rest of the combustion chamber. Throughout this work it will be assumed that all heat absorption by the

INTRODUCTION

According to C. I. Adler (1), the preparation of the base  
for the bisacetyl ester is as follows: The bisacetyl ester is  
obtained from the bisacetone ester by the following  
process: The bisacetone ester is dissolved in ether and  
then the ether solution is added to a solution of  
potassium hydroxide in water. The ether is then  
distilled off and the residue is washed with  
water and dried. The product is a white  
solid with a melting point of 100°C. It is  
soluble in ether, chloroform, and benzene  
but insoluble in water. The yield is about  
70%.

piston takes place by convection. Also all heat dissipated by the piston will be assumed to do so by a process of convection.

The temperature of the gases throughout the cycle varies widely; the temperature curves for an engine may be obtained either experimentally by direct measurement or analytically from knowledge of the cycle under which the engine is operating. In this work the temperature of the gases will be assumed, for the sake of simplicity, to follow a sinusoidal variation about an average temperature.

The main objective of this work will be to determine the temperature distribution throughout the piston. For this purpose a set of boundary conditions will be prescribed which resemble as closely as possible the actual conditions under which the piston operates.

the piston please give a concession. After all the piston is being  
used to a process of concession.

The member states of the base price group are  
under which the base price is observed.

## CHAPTER II

### STATEMENT OF THE PROBLEM

#### 1. Description of piston.

A piston with the characteristics shown in figure 1 was selected for analysis. The crown will be a flat disk of uniform thickness  $l$ . The barrel will be a cylindrical sleeve of constant thickness  $b$ .

The boundary surface of disk and barrel will be surface a-a as shown in figure 1. This surface will have properties common to both disk and barrel and this will enable some of the unknown quantities entering the questions to be found.

#### 2. Temperature scales.

Figure 2 shows a sketch of the temperature pattern that a particular point of the disk is expected to follow. This figure is given mainly to show graphically some of the values being used throughout the development.

Figure 3 serves the same purpose as figure 2 with relation to the barrel.

The temperature of the gases will be assumed to be:

$$T_{gas} = T_m + T_o \sin 2\pi n \tau$$

where  $n$  will be the frequency of the cycle.

The temperature  $T_a$  surrounding the outside surface of the barrel will be assumed constant.

The temperatures for the disk will be measured from the level designated as in figure 2 and will be expressed

## STATEMENT OF THE PROBLEM

1. Description of system.

A piston with the characteristics of Figure 1 was selected for analysis. The piston will be assumed to be cylindrical and perfectly elastic. The following properties of the piston are assumed.

The fundamental frequency of vibration of the piston will be assumed to be 8-8 as shown in Figure 1. This frequency will be proportional to the mass of the piston and will be independent of the dimensions of the piston. The following properties of the piston will be assumed to be constant.

2. Temperature coefficients.

Figure 2 shows a specimen of the membrane before heating. This is a rectangular portion of the skin as expected of leather. The specimen is shown in Figure 3 after being heated to a temperature of about 140° F. This figure is given mainly to show the effect of the heating.

Figure 3 shows the same portion as in Figure 2 with the following changes.

The membrane of the base will be assumed to be:

$$T_{\text{new}} = T + \alpha T = T(1 + \alpha)$$

where  $\alpha$  will be the modulus of the object. The membrane is to undergo the following changes in the surface of the piston will be assumed to be:

The membrane for the air will be assumed to be:

The level measured as in Figure 2 and will be assumed

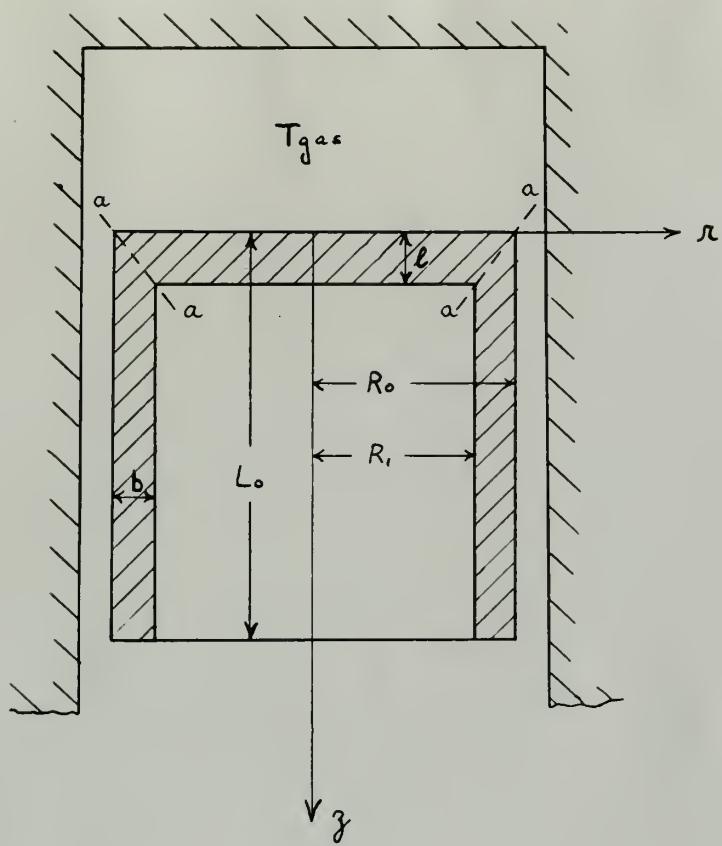


Figure 1



for any point of the disk as:

$$\theta = t - T_m$$

The initial temperature will necessarily be a constant and will be designated as  $\theta_0$ .

The temperatures for points of the barrel will be measured in the  $\phi$  scale of temperatures as shown in figure 3, where:

$$\phi = t - T_a$$

The initial temperature  $\phi_0$  will be a constant.

It is realized that the value of  $T_a$  will initially be equal to  $t_0$ , but it is impossible with any degree of ease to consider into the problem its initial rise from  $t_0$  to  $T_a$  and thus it will be assumed constant and equal to  $T_a$ .

After a solution is reached, the results will be reverted to the standard level of measurement of temperatures in degrees F. which is designated as the  $t$  scale of temperatures.

### 3. Boundary conditions.

The following set of boundary conditions has been selected:

(a). At the lower surface of the disk, the rate of heat rejection is ten percent of the rate of heat absorption at the upper surface. Since the rate of heat conduction is a function of the temperature gradient normal to the surface considered, and if we assume the coefficient of heat conduction to be constant throughout the piston, we may state:

for the body to the air to

$$\theta = \tau - \tau_0$$

The initial temperature will decrease as a consequence

and will be deflected as  $\theta$ .

The members will be lost to the boundary as shown in figure  
measuring in the  $\phi$  sense of  $\theta$  to the  $\theta$  sense of  $\tau - \tau_0$ .

3. Above:

$$\theta = \tau - \tau_0$$

If it is assumed that the value of  $\theta$  will be less than that of  $\tau - \tau_0$  then it is possible with the same degree of loss

to consider that the boundary condition is to be imposed on the boundary of the domain.

After a solution is obtained, the boundary will be

referred to the steady state of members of boundary and the temperature of the boundary will be assumed as the same as the steady state of the members.

3. Boundary conditions.

The following set of boundary conditions are given

set forth:

(a). At the lower surface of the plate, the rate of heat conduction is a function of the rate of heat convection and the upper surface. Since the rate of heat convection is at the upper surface, the condition of the boundary is a function of the temperature gradient existing mostly of the boundary condition, and if we assume the condition of the boundary to be constant throughout the duration, we may state:

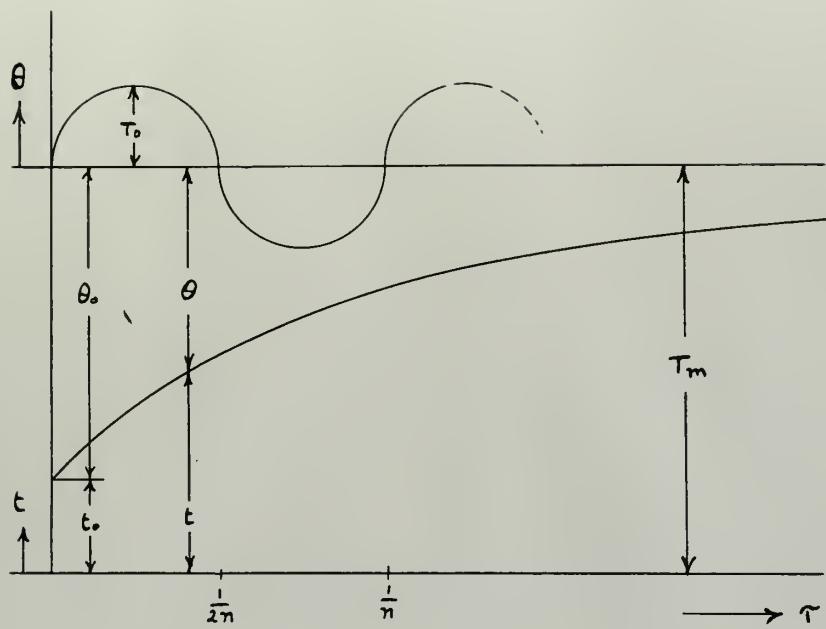


Figure 2



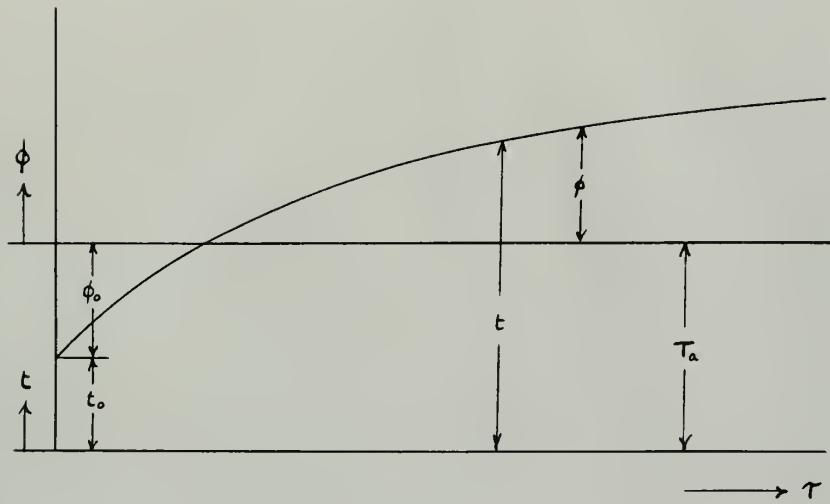


Figure 3



$$\left(\frac{\partial \theta}{\partial z}\right)_{z=0} = 10 \left(\frac{\partial \theta}{\partial z}\right)_{z=\epsilon}$$

There will be a difference in the areas of the upper and lower surfaces determined by the relative values of  $R_o$  and  $R_i$ . In this work we will neglect this difference, but it is recognized that it could be included by multiplying the value given as 10 by the ratio of  $R_i$  to  $R_o$  squared.

(b). At the upper surface of the disk the rate of heat absorption from the gases will equal the rate of heat conduction from the surface into the disk, or:

$$k \left(\frac{\partial \theta}{\partial z}\right)_{z=0} = -h_i (\theta_{gas} - \theta_{z=0})$$

(c). At the surface a-a the temperature of both the disk and the barrel will be equal. Since these temperatures will be measured from different levels, this relation may not be used directly without introducing undesirable constants.

Therefore it is necessary to realize that at this boundary the isotherms from disk and barrel must coincide and furthermore they must be continuous functions of the variables involved. This may be stated symbolically: along surface a-a:

$$\frac{\partial \theta}{\partial z} = \frac{\partial \phi}{\partial z}$$

and:

$$\frac{\partial \theta}{\partial n} = \frac{\partial \phi}{\partial n}$$

$$\therefore \left( \frac{\theta}{L} \right)_{\text{at}} = \left( \frac{\theta}{L} \right)_{\text{at}}$$

There will be a difference in the area of the upper and lower surface determined by the relative values of  $R_0$  and  $R_1$ . In this work  $R_0$  will neglect this difference but it is necessary that it could be imagined a unit playing the value given as 10 for the ratio of  $R_1$  to  $R_0$  satisfied.

(d). At the upper surface of the disk the rate of heat absorption from the base will equal the rate of heat conduction from the surface into the disk, or:

$$( \theta - \theta_{\text{at}} )_{\text{at}} = \left( \frac{\theta}{L} \right)_{\text{at}}$$

(e). At the surface  $\theta = \theta_{\text{at}}$  the temperature of both the disk and the part will be equal. Since this temperature will be measured from different levels, this relation may not be used directly without introducing undesirable constants.

Therefore if it is necessary to relate  $\theta$  to  $\theta_{\text{at}}$  it is necessary to suppose that  $\theta$  and  $\theta_{\text{at}}$  are continuous from disk and part with continuous functions of the same temperature type must be continuous functions of the variables involved. This may be stated approximately as follows:

$$\frac{\theta''L}{\pi rL} = \frac{\theta''L}{\pi rL}$$

$$\frac{\theta''L}{\pi rL} = \frac{\theta''L}{\pi rL}$$

and

(d). The lower surface of the barrel will be assumed insulated, or:

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=L_0} = 0$$

(e). The inner surface of the barrel will be assumed insulated, or:

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=R_1} = 0$$

(f). At the outer surface of the barrel the rate of heat rejection from the barrel into the surrounding medium will be equal to the rate of heat conduction from the barrel into its outer surface, or:

$$k \left(\frac{\partial \phi}{\partial r}\right)_{r=R_0} = -h_3 (\phi_{r=R_0} - \phi_a)$$

(g). At time = zero, a point:  $z = 0$ ,  $r = 0$ , will be at a temperature  $\theta_0$ , or:

at

$$\tau = 0 \quad r = 0 \quad z = 0 \quad \theta = \theta_0$$

the lower surface will be assumed to be perfectly insulated. (b)

transferring, or:

$$0 = \left\{ \begin{array}{l} \left( \frac{\phi_L}{L} \right) \\ \left( \frac{\phi_L}{L} \right) \end{array} \right.$$

the inner surface of the parallel will be assumed to be perfectly insulated. (c)

transferring, or:

$$0 = \left\{ \begin{array}{l} \left( \frac{\phi_L}{L} \right) \\ \left( \frac{\phi_L}{L} \right) \end{array} \right.$$

At the outer surface of the parallel the rate of

heat rejection from the parallel into the surroundings will be equal to the rate of heat conduction from the

parallel into the outer air, or:

$$(\phi - \phi_{\infty})_{\text{ext}} = \left( \frac{\phi_L}{L} \right) \times$$

At  $\phi = 0$ ,  $\phi = 2$ ,  $\phi = 5$ ,  $\phi = 7$ ,  $\phi = 9$ ,  $\phi = 10$ ,  $\phi = 12$ . (g)

as a temperature  $\theta$ , or:

$$\theta = \theta$$

$$\theta = 6 \quad \theta = 5 \quad \theta = 7 \quad \theta = 9$$

## CHAPTER III

### SOLUTION OF PROBLEM

#### 1. General solution.

In solving the problem the disk and barrel will be treated separately and the solutions matched at the surface a-a.

Both the disk and the barrel involve heat transfer in a cylinder; the disk may be considered as a flat solid cylinder, while the barrel may be considered as a hollow cylinder. Under these conditions the best solution of Fourier's Law of Conduction of Heat will be that expressed in cylindrical coordinates:

$$\frac{dt}{tr} = \alpha \left( \frac{r^2 t}{J_1^2} + \frac{1}{r} \frac{dt}{J_1} + \frac{r^2 t}{J_3^2} \right) \quad (1)$$

Assuming the variables to be separable a solution as found in Appendix I is:

$$t \cdot E e^{-\alpha(a^2+b^2)r} [C \cos b_j + D \sin b_j] [A J_0(ar) + B Y_0(ar)] \quad (2)$$

From figure 2 it may be seen that the expected curves for temperatures will be of the form:

$$\theta = \theta_0 e^{-\alpha r}$$

for a particular point on the disk.

At the same time a sinusoidal variation will be applied, which proceeding along the z-axis will be damped and out of phase; therefore our solution would have a form such as:

## SOLUTION OF PROBLEMS

## J. General Solution.

In solving the problem the disk and paraboloid will be treated separately and the solutions matching at the surface

.8-8

Both the disk and the paraboloid involve first parameter in

a cylinder; the disk may be considered as a flat solid cylinder, while the paraboloid may be considered as a hollow cylinder. Under these conditions the first solution of Fourier's Law of Conduction of Heat will be first expressed in cylindrical coordinates:

$$(1) \quad \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0$$

Assuming the axisipal to be separable a solution as

found in Appendix I is

$$(2) \quad \left[ (r) A \delta + (r) B \right] \left[ (r) C \cos \theta + (r) D \sin \theta \right] e^{-\alpha(r^2+z^2)}$$

From figure 2 it will be seen that the expected curve

for temperatures will be of the form:

$$e^{-\alpha(r^2+z^2)} \theta = \theta$$

for a cylindrical boundary on the disk.

At the same time a sinusoidal variation will be obtained in the angle about the z-axis will be obtained along the z-axis and the principal proceeding along the z-axis only have a form such as:

$$\theta = \theta_0 e^{-az} + T_0 \sin(bz - c_3) \cdot e^{-dz}$$

where  $c_3$  is the phase angle and  $e^{-dz}$  accounts for damping.

Examining the previous equation it may be seen that the variables  $z$  and  $\tau$  may not be separated as assumed in the solution found by equation (2), and thus this equation will not yield an exact solution.

A direct solution of the problem was attempted by the use of Duhamel's Theorem (3), but it was found impossible to match the result obtained for the disk with that for the barrel at the surface  $a-a$ .

Under these conditions it was decided to divide the problem into one where the gas temperature was assumed to be the average temperature  $T_m$ , and then proceed to superimpose on it the effect of the sinusoidal variation in gas temperature.

## 2. Solution for the disk.

From equation (2):

$$\theta = E e^{-\alpha(a^2 + b^2)\tau} [C \cos b_3 + D \sin b_3] [A J_0(a z) + B Y_0(a z)]$$

Since  $\theta$  must be finite throughout the body:

$$B = 0$$

and therefore:

$$\theta = F e^{-\alpha(a^2 + b^2)\tau} [C \cos b_3 + D \sin b_3] J_0(a z) \quad (3)$$

From boundary condition (a):

$$\left( \frac{\partial \theta}{\partial z} \right)_{z=0} = 10 \left( \frac{\partial \theta}{\partial z} \right)_{z=l}$$

$$e^{-\theta} \cdot e^{-\theta} \cdot (1 - e^{-\theta}) \sin \theta + e^{-\theta} \cdot e^{-\theta} \cdot \theta = 0$$

where  $\theta$  is the price subjective and  $e^{-\theta}$  is the second price for assembly. Examining the derivative of  $\theta$  may be seen first the asymptotes and  $\theta$  may not be separated as assumed in the solution. From the definition (2), and this is the case, the solution is not a solution of the equation  $\theta = 0$ .

A direct solution of the problem is a sufficient proof of the use of Duhesme's Theorem (3), but it was found impossible to match the result of Duhesme's Theorem (3) with the result of the first for the part of the surface  $\theta = 0$ .

Under these conditions it is decided to divide the problem into one where the gas members are assumed to be the average members, i.e. the only pure process of superimpose on it if the effect of the same is negligible.

2. Solution for the other.

From definition (2):

$$\left[ (1 - e^{-\theta}) \left[ e^{-\theta} \sin \theta + e^{-\theta} \cos \theta \right] + e^{-\theta} \right] \cdot \left[ e^{-\theta} \sin \theta + e^{-\theta} \cos \theta \right] = 0$$

Since  $\theta$  must be finite provided the logy:

$$0 = E$$

and therefore:

$$(3) \quad \left[ (1 - e^{-\theta}) \left[ e^{-\theta} \sin \theta + e^{-\theta} \cos \theta \right] + e^{-\theta} \right] \cdot \left[ e^{-\theta} \sin \theta + e^{-\theta} \cos \theta \right] = 0$$

From previous condition (2):

$$1 - \left( \frac{\theta}{L} \right) = 10 \left( \frac{\theta}{L} \right)$$

From equation (3):

$$\frac{J\theta}{J_3} = Fe^{-\alpha(a^2+b^2)^{1/2}} \left[ -C_6 \sin b_3 + D_6 \cos b_3 \right] J_0(ar)$$

therefore:

$$\frac{D}{C} = \frac{\sin bl}{\cos bl - 0.1}$$

then:

$$\theta = G e^{-\alpha(a^2+b^2)^{1/2}} \left[ \cos b_3 + \frac{\sin bl}{\cos bl - 0.1} \sin b_3 \right] J_0(ar) \quad (4)$$

From boundary condition (b):

$$k \left( \frac{J\theta}{J_3} \right)_{z=0} = -h_i (\theta_{z=0} - \theta_{z=0})$$

or:

$$\frac{k}{h_i} \left( \frac{J\theta}{J_3} \right)_{z=0} = \theta_{z=0}$$

then:

$$\frac{\sin bl}{\cos bl - 0.1} = \frac{h_i}{k b}$$

The solution of this equation for b may only be done graphically; it will yield a series of values for b which will be designated as  $b_m$ .

Equation (4) now becomes:

$$\theta = \sum_{m=1}^{\infty} G e^{-\alpha(a^2+b_m^2)^{1/2}} \left[ \cos b_m z + \frac{h_i}{kb_m} \sin b_m z \right] J_0(ar) \quad (5)$$

From boundary condition (g):

$$\tau = 0 \quad z = 0 \quad r = 0 \quad \theta = \theta_0$$

Substituting this in equation (5):

$$G = \theta_0$$

:(3) solution sought

$$(s\omega)_0 L \left[ \left( \frac{d}{dt} \cos \theta + \frac{d}{dt} \sin \theta \right) \right] \tau^{(d+1)\omega} - \omega^2 = \frac{\partial L}{\partial t}$$

equation:

$$\frac{d \sin \theta}{1 - \omega^2 \sin^2 \theta} = \frac{\omega}{L}$$

then

$$(s\omega)_0 L \left[ \left( \frac{d \sin \theta}{1 - \omega^2 \sin^2 \theta} + \frac{d \cos \theta}{1 - \omega^2 \cos^2 \theta} \right) \right] \tau^{(d+1)\omega} - \omega^2 = 0$$

(4)

:(4) solution sought

$$(s\omega_1 \theta - \omega_0 \theta)_{,1} - = \left( \frac{\partial L}{\partial \dot{\theta}} \right) \dot{\theta}$$

$$s\omega_1 \dot{\theta} = \left( \frac{\partial L}{\partial \dot{\theta}} \right) \dot{\theta}$$

$$\frac{d \dot{\theta}}{d \theta} = \frac{2 \omega_0 \omega}{1 - \omega^2 \sin^2 \theta}$$

and as this is not a solution with  
definite and easily to solve a likely will it be difficult  
solutions with (1) and  
as before, take the  
solutions with (2) and

$$(s\omega)_0 L \left[ \left( \frac{d \sin \theta}{1 - \omega^2 \sin^2 \theta} + \frac{d \cos \theta}{1 - \omega^2 \cos^2 \theta} \right) \right] \tau^{(d+1)\omega} - \omega^2 \sum_{n=1}^{\infty} = 0$$

:(5) solution sought

$$\omega_0 \theta = \theta \quad \theta = \omega_0 t \quad \theta = \omega_0 t \quad \theta = \omega_0 t$$

:(6) solution of initial conditions

$$\theta = \omega_0 t$$

Therefore:

$$\theta = \sum_{m=1}^{\infty} \theta_m e^{-\omega(a^2+b_m^2)\tau} \left[ \cos b_m z + \frac{h_1}{k b_m} \sin b_m z \right] J_0(a z) \quad (6)$$

3. Solution for the barrel.

From Equation (2):

$$\phi = M e^{-\omega(f^2+g^2)\tau} \left[ \cos g z + L \sin g z \right] \left[ R J_0(f z) + N Y_0(f z) \right] \quad (7)$$

From boundary condition (d):

$$\left( \frac{\partial \phi}{\partial z} \right)_{z=L_0} = 0$$

therefore:

$$L = \tan g L_0$$

then:

$$\phi = P e^{-\omega(f^2+g^2)\tau} \cos g(z-L_0) \left[ R J_0(f z) + N Y_0(f z) \right] \quad (8)$$

From boundary condition (e):

$$\left( \frac{\partial \phi}{\partial z} \right)_{z=R_1} = 0$$

or:

$$\frac{N}{R} = - \frac{J_1(f R_1)}{Y_1(f R_1)} = F$$

therefore:

$$\phi = Q e^{-\omega(f^2+g^2)\tau} \cos g(z-L_0) \left[ J_0(f z) + F Y_0(f z) \right] \quad (9)$$

From boundary condition (f):

$$k \left( \frac{\partial \phi}{\partial z} \right)_{z=R_0} = - h_3 (\phi)_{z=R_0} = 0$$

From superposition:

$$(d) \quad (s2) \circ \bar{I} \left[ \text{cosec} \frac{1}{\omega} \theta + \{ \omega \} \right]^{\tau(1, \beta + \delta)} \circ \theta \sum_{n=1}^{\infty} = \emptyset$$

3. Solution for the parallel.

From superposition:

$$(e) \quad [(s1) \circ M + (s2) \circ \bar{R}] \left[ \text{cosec} \theta + \{ \omega \} \right]^{\tau(1, \beta + \delta)} \circ \theta M = \emptyset$$

From parallel condition:

$$0 = \begin{pmatrix} \frac{dL}{d} \\ \frac{d}{d} \end{pmatrix}$$

From parallel:

$$\omega \circ \text{mat} = 1$$

From:

$$(e) \quad [(s1) \circ M + (s2) \circ \bar{R}] (0, 1, \{ \omega \})^{\omega} \circ \theta M = \tau(1, \beta + \delta) \circ \theta M = \emptyset$$

From parallel condition:

$$0 = \begin{pmatrix} \frac{dL}{d} \\ \frac{d}{d} \end{pmatrix}$$

$$0 = \frac{(s1) \circ L}{(s2) \circ R} = \frac{M}{R}$$

From parallel:

$$(e) \quad [(s1) \circ \bar{R} + (s2) \circ \bar{I}] (0, 1, \{ \omega \})^{\omega} \circ \theta M = \tau(1, \beta + \delta) \circ \theta M = \emptyset$$

From parallel condition:

$$(0, 1, \{ \omega \})^{\omega} = \begin{pmatrix} \frac{dL}{d} \\ \frac{d}{d} \end{pmatrix} *$$

or: 
$$\frac{f k}{h_3} = \frac{J_0(f R_0) + F Y_0(f R_0)}{J_1(f R_0) + F Y_1(f R_0)}$$

Solving this equation graphically for the value of  $f$ , a set of values  $f_p$  are found that satisfy this equation, and equation (9) becomes:

$$\phi = \sum_{p=1}^{\infty} P e^{-\alpha(f_p^2 + g^2) \tau} \cos g(z - L_0) [J_0(f_p z) + F_p Y_0(f_p z)] \quad (10)$$

#### 4. Conditions at the surface a-a.

Recapitulating:

$$\theta = \sum_{m=1}^{\infty} \theta_m e^{-\alpha(a^2 + b_m^2) \tau} \left[ \cos b_m z + \frac{h_1}{k b_m} \sin b_m z \right] J_0(a z) \quad (6)$$

and:

$$\phi = \sum_{p=1}^{\infty} P e^{-\alpha(f_p^2 + g^2) \tau} \cos g(z - L_0) [J_0(f_p z) + F_p Y_0(f_p z)] \quad (10)$$

From boundary condition (c) at points along the surface a-a:

$$\frac{\partial \theta}{\partial z} = \frac{\partial \phi}{\partial z} \quad (11)$$

$$\frac{\partial \theta}{\partial z} = \frac{\partial \phi}{\partial z} \quad (12)$$

For equation (11) it is easily seen that for  $n=1$  and for a particular point along the surface a-a:

$$\sum_{m=1}^{\infty} T_m = \sum_{p=1}^{\infty} S_p \quad (13)$$

then for  $n=5$ :

$$\sum_{m=1}^{\infty} b_m^4 T_m = \sum_{p=1}^{\infty} g^4 S_p \quad (14)$$

this suggests that as a method of approach to the evaluation of  $a$ ,  $g$ , and  $P$  we assume:

$$g = b_m$$

$$\frac{(0.8)(A\pi + (0.8)(0.7))}{(0.8)(A\pi + (0.8)(0.7))} = \frac{A\pi}{0.7}$$

3. To solve this equation for  $A\pi$  we have to take a set of values for  $A\pi$  and  $0.7$  that gives a solution and  $A\pi$  becomes:

$$(11) \quad \left[ (0.8)(A\pi + (0.8)(0.7)) (0.7 - 0.8) \right] \sum_{i=0}^{\infty} 7^{i+1} 0.7^i = 0.7 \sum_{i=0}^{\infty} 0.7^i = 0$$

4. Conditions of the solution are:

Decomposition:

$$(12) \quad 0.7 \left[ \left( \text{and } \frac{0.7}{0.7} + \left( \text{and } \frac{0.7}{0.7} \right) \right) \sum_{i=0}^{\infty} 7^{i+1} 0.7^i - 0.7 \sum_{i=0}^{\infty} 0.7^i \right] = 0$$

and:

$$(13) \quad \left[ (0.8)(A\pi + (0.8)(0.7)) (0.7 - 0.8) \right] \sum_{i=0}^{\infty} 7^{i+1} 0.7^i - 0.7 \sum_{i=0}^{\infty} 0.7^i = 0$$

From previous equation (13) we can find the

surface area:

$$(14) \quad \frac{0.7}{0.7} = \frac{0.7}{0.7}$$

$$\frac{0.7}{0.7} = \frac{0.7}{0.7}$$

For condition (14) we have to take a

and for a better understanding we can take the values of:

$$(15) \quad 0.7 \sum_{i=0}^{\infty} = 0.7 \sum_{i=0}^{\infty}$$

then for  $i = 0$ :

$$(16) \quad 0.7 \sum_{i=0}^{\infty} = 0.7 \sum_{i=0}^{\infty}$$

and we can see a mapping of the function to the

expression of  $a$ ,  $b$ ,  $c$ , and  $D$  is same:

Acceptance of this assumption involves the necessity, in order to maintain the equality of the above equation, that:

$$T_m = S_p$$

where:

$$m = p$$

Also, in order that the effect of time on the temperature be equal for the same point on the surface a-a as approached by either equation (6) or (10), that:

$$f_p = a$$

Applying this condition to equation (13), we find:

$$\theta_0 \left[ \sin b_m \beta - \frac{\lambda_1}{kb_m} \cos b_m \beta \right] = P \sin b_m (\beta - L_0) \left[ 1 - \frac{J_1(f_p R_1) Y_0(f_p a)}{Y_1(f_p R_1) J_0(f_p a)} \right] \quad (15)$$

where:

$$m = p$$

This equation is valid for points along the surface a-a, where:

$$\beta = \frac{\lambda}{b} (R_0 - r)$$

and should yield the values of  $P_p$ .

Now, the replacement of equation (15) into equation (14) will yield an equation in terms of trigonometric and Bessel functions of  $r$ , valid for values of  $r$  ranging from  $R_1$  to  $R_0$ . Since this equation may not be simplified to any appreciable extent, it has been considered more advisable to replace the values of  $z$  and  $r$  of a particular point along the surface a-a, such that:

$$z = \lambda/2 = \lambda_a$$

$$r = (R_1 + R_0)/2 = R_a$$

Accomplishment of this construction involves the necessary

in order to make up the space demanded by the definition

that:

$$q^2 = m^2$$

$$q = m$$

where:

Also, in order that the difference of time on the members

must be equal to the time on the body on the surface  $s-a$  as

approached by either direction (a) or (b) that:

$$s = q^1$$

: using this condition (13), we find

$$(13) \quad \left[ \frac{(s-a) \sqrt{g_1 g_2 g_3}}{K} - 1 \right] (a - s) \sin \theta = \left[ g_1 s \frac{d}{ds} - g_2 s \right] \theta$$

where:

$$q = m$$

This defines the value for points along the surface

$$(a-s) \frac{d}{ds} = \theta$$

s-a, where:

and showing also the values of

Now, the replacement of condition (12) into the definition

will yield an expression of the form of (14) with

based functions of  $s$ , valid for values of  $s$  ranging from

0 to  $R_0$ . Since this condition may not be satisfied for

applicable except if the path considered more closely

to replace the value of  $s$  in a point of a spherical point

along the surface  $s-a$ , such that:

$$s\lambda = s/\lambda = s$$

$$\lambda = s/(m + p) = q$$

then we find:

$$\theta_0 \left[ \sin b_m l_a - \frac{h_1}{k b_m} \cos b_m l_a \right] = P_p \sin b_m (l_a - L_0) \left[ 1 - \frac{J_1(f_p R_1) Y_0(f_p R_a)}{Y_1(f_p R_1) J_0(f_p R_a)} \right] \quad (16)$$

where:

$$m = p$$

If now in equation (12) for  $n = 1$  this same point in the surface a-a is investigated:

$$\theta_0 \left[ \cos b_m l_a - \frac{h_1}{k b_m} \sin b_m l_a \right] = P_p \cos b_m (l_a - L_0) \left[ 1 - \frac{J_1(f_p R_1) Y_1(f_p R_a)}{Y_1(f_p R_1) J_1(f_p R_a)} \right] \quad (17)$$

Equations (16) and (17) should give the same value for  $P_p$ . There is no way of proving that this will be so except by working out a particular problem. If in so doing it is found that the values of  $P_p$  satisfy simultaneously both equations, this will then mean that the assumption made that:

$$g = b_m$$

and the derived expressions:

$$T_m = S_p$$

and:

$$f_p = a$$

are justified.

In case that the values of  $P_p$  found by means of one of the equations mentioned do not satisfy the other equation, there is still the possibility of introducing this value in the second equation and determining a relationship between  $R_1$ ,  $b$ ,  $L_0$ , and  $l$  such that both equations are satisfied and for which the assumption made is valid. This would introduce a limitation to the physical proportions of the pistons for which this development is useful. The relationship just mentioned would take the form:

$$(21) \quad \left[ \frac{(\lambda \Delta_0) \otimes (\lambda \Delta_0) \otimes \lambda}{(\lambda \Delta_0) \otimes \lambda (\lambda \Delta_0) \otimes \lambda} - 1 \right] (\lambda - \lambda) \text{ and } \text{min } g_F = \left[ \lambda - \lambda \cos \frac{\pi k}{n+1} - \lambda \sin \text{min} \right] \otimes$$

beverage is a soft drink.

$$(71) \quad \left[ \frac{(\omega_0)(\lambda)(\alpha_0))\pi}{(\omega_0)(\lambda)(\alpha_0))\pi} - 1 \right] (\omega - \omega_0) \sin \theta = \left[ \omega_0 \sin \frac{\omega_0}{\omega_0 + \omega} - \omega_0 \sin \omega \right] \theta$$

Estimations (TE) and (TE) found give the same value

for  $P_b$ . There is no way to remove such pairs until we do

except a writing out a definition blog. It is no going

If it is found that the avianes of the *Pyrrhuloxia* simulating *Amphispiza*

Some examples of this are the following:

$m^d = 3$  :  $\text{f} \text{a} \text{d} \text{f}$

and the derived extra factors.

$$a^2 = m^2$$

$$s = g^2 : bns$$

• *beautified* *the*

In case that the value of the function  $\varphi$  measure of one

terro ed visita Jon ob hær fyrre enitwæp eni fo

and the other to the post office at the same time.

This is a file in the second edition using the new definition of *adhesives*.

After these days of rest, I had the opportunity to go to the beach.

education sites specifically designed for the study of the natural environment.

top/s out at soft tit. If a seafloor below slot - bit/sy at

at the top of each block, with the soft switch set to `softswitch`.

Digitized by srujanika@gmail.com

$$\frac{\sin b_m \ln - \frac{h_i}{k b_m} \cos b_m \ln}{\cos b_m \ln + \frac{h_i}{k b_m} \sin b_m \ln} = \tan b_m (\ln - L_0) \frac{1 - \frac{J_1(f_p R_i) Y_0(f_p R_a)}{Y_1(f_p R_i) J_0(f_p R_a)}}{1 - \frac{J_1(f_p R_i) Y_1(f_p R_a)}{Y_1(f_p R_i) J_1(f_p R_a)}} \quad (18)$$

### 5. Partial solution using $T_m$ .

On the light of previous conclusions, the solution is

now:

$$\theta = \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \theta_0 e^{-\alpha(f_p^2 + b_m^2)\tau} \left[ \cos b_m \ln + \frac{h_i}{k b_m} \sin b_m \ln \right] J_0(f_p \ln) \quad (19)$$

$$\phi = \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} P_p e^{-\alpha(f_p^2 + b_m^2)\tau} \cos b_m (\ln - L_0) \left[ J_0(f_p \ln) + F_p Y_0(f_p \ln) \right] \quad (20)$$

and since:

$$\theta = t - T_{gas}$$

and:

$$\phi = t - T_a$$

we find, for the disk:

$$t = T_{gas} + \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \theta_0 e^{-\alpha(f_p^2 + b_m^2)\tau} \left[ \cos b_m \ln + \frac{h_i}{k b_m} \sin b_m \ln \right] J_0(f_p \ln) \quad (21)$$

and for the barrel:

$$t = T_a + \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} P_p e^{-\alpha(f_p^2 + b_m^2)\tau} \cos b_m (\ln - L_0) \left[ J_0(f_p \ln) + F_p Y_0(f_p \ln) \right] \quad (22)$$

### 6. Superposition of sinusoidal variation.

In order to superimpose the effect of the sinusoidal fluctuation of the gas temperature about a mean temperature  $T_m$ , the above equation (21) in a simplified form:

$$t = T_{gas} + f(r, \ln, \tau)$$

$$(81) \quad \frac{\frac{(\omega_1 t) \mathcal{Y}(\omega_1 t)}{\mathcal{L}(\omega_1 t) \mathcal{Z}(\omega_1 t)}}{\frac{(\omega_2 t) \mathcal{Y}(\omega_2 t)}{\mathcal{L}(\omega_2 t) \mathcal{Z}(\omega_2 t)}} = 1 \quad (\omega_1 - \omega_2) \text{ and not} = \frac{\omega_1 \text{ and not } \frac{d}{dt} - \omega_2 \text{ and not}}{\omega_2 \text{ and not } \frac{d}{dt} + \omega_1 \text{ and not}}$$

the value of  $\omega_1$  and  $\omega_2$  is

•  $\omega_1$  and  $\omega_2$  are called natural frequencies

so the solution is

$$(e1) \quad (\omega_1 t) \mathcal{Z} \left[ \omega_1 \text{ and not } \frac{d}{dt} + \omega_1 \text{ and not} \right] \mathcal{Y}^{(\omega_1 t + \omega_2 t)} \rightarrow 0.8 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} = 0$$

$$(e2) \quad \left[ (\omega_1 t) \mathcal{Y} \mathcal{Z} + (\omega_2 t) \mathcal{Z} \right] (\omega_1 - \omega_2) \text{ and not} \mathcal{Y}^{(\omega_1 t + \omega_2 t)} \rightarrow 0.7 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} = \phi$$

$$\omega_1 t - \omega_2 t = \theta$$

: same basis

$$\omega t - \omega t = \phi$$

: basis

: half and not half

$$(f1) \quad (\omega_1 t) \mathcal{Z} \left[ \omega_1 \text{ and not } \frac{d}{dt} + \omega_1 \text{ and not} \right] \mathcal{Y}^{(\omega_1 t + \omega_2 t)} \rightarrow 0.8 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} + \omega_1 t = \theta$$

: forced and not basis

$$(f2) \quad \left[ (\omega_1 t) \mathcal{Y} \mathcal{Z} + (\omega_2 t) \mathcal{Z} \right] (\omega_1 - \omega_2) \text{ and not} \mathcal{Y}^{(\omega_1 t + \omega_2 t)} \rightarrow 0.7 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} + \omega_2 t = \theta$$

• a free vibration is a solution to a differential equation

In order to obtain this we use the

initial conditions as a free vibration is a solution to a differential equation

: not a solution is at (f2) no longer valid

$$(\mathcal{Y}(\omega_1 t) + \omega_1 t) + \omega_1 t = \theta$$

after a quarter of a cycle, or at  $\tau = \frac{1}{4n}$  we find that:

$$T_{gas} = T_m + T_0 \sin 2\pi n \tau$$

therefore:

$$T_{gas} = T_m + T_0$$

If this were put into equation (21) it would give an answer corresponding to the situation where this new temperature would have been imposed on the piston since time zero.

To avoid this, instead of:

$$T_0 \sin 2\pi n \tau \quad (23)$$

use the following expression given by M. Jakob and G. A. Hawkins (7):

$$T_0 e^{-3\sqrt{\frac{\pi n}{\alpha}}} \sin \left( 2\pi n \tau - 3\sqrt{\frac{\pi n}{\alpha}} \right) \quad (24)$$

where:

$T_0$  is the amplitude of the oscillation at the surface of the disk;

$e^{-3\sqrt{\frac{\pi n}{\alpha}}}$  accounts for damping effects along  $z$ , and

$-3\sqrt{\frac{\pi n}{\alpha}}$  accounts for the change in phase along  $z$ .

The above expression was given for a thick plate subjected to conditions similar to those of this problem. Even if in this case the disk may perhaps not be considered a thick plate, it is felt that this equation will yield a fairly good approximation if it is considered that it is common knowledge that the effects of fluctuating surface temperatures do not penetrate to any great extent beyond the surface exposed. In this particular case this effect is further minimized by the fact that the fluctuation of temperatures is very rapid.

$$\begin{aligned}
 \text{Let } T_0 \text{ be the initial temperature.} \\
 T_0 + \alpha T + \beta T^2 = T_0 e^{\gamma T} \\
 \alpha T + \beta T^2 = e^{\gamma T} - 1
 \end{aligned}$$

as we are trying to find  $T$  in terms of  $\alpha$ ,  $\beta$ , and  $\gamma$ .

we can rearrange the equation to get  $T$  on one side of the equation.

(28)

$$T_0 + \alpha T + \beta T^2 = e^{\gamma T}$$

Now we can solve this equation for  $T$ .

(29)

$$(T_0 + \alpha T + \beta T^2) = e^{\gamma T}$$

Now we can solve this equation for  $T$ .

The good news is that we can use the quadratic formula to solve this equation. The quadratic formula is given by  $T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . In this case,  $a = \beta$ ,  $b = \alpha$ , and  $c = T_0$ . Substituting these values into the quadratic formula, we get  $T = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta T_0}}{2\beta}$ . This is the final answer.

From equations (23) and (24) it may be seen that what happens at the surface at time zero will have its effect at a depth  $z$  with a time lag of:

$$\Delta \tau = \frac{1}{2} z \sqrt{\frac{1}{\alpha \pi n}}$$

The addition of the term expressed in equation (24) will affect in equal form both the equations (21) and (22). Since for the barrel the values of  $z$  will, with the exception of its topmost part, be far greater than those for the disk, this effect will only be perceptible in its upper section.

Including equation (24) into equations (21) and (22) we have as a final result:

For the disk:

$$t = T_m + T_0 e^{-3 \sqrt{\frac{\pi n}{\alpha}} \tau} \sin(\omega \pi n \tau - 3 \sqrt{\frac{\pi n}{\alpha}}) + \\ + \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \theta_0 e^{-\alpha (f_p^2 + b_m^2) \tau} \left[ \cos b_m \beta + \frac{h_1}{k b_m} \sin b_m \beta \right] J_0(f_p r)$$

For the barrel:

$$t = T_a + T_0 e^{-3 \sqrt{\frac{\pi n}{\alpha}} \tau} \sin(\omega \pi n \tau - 3 \sqrt{\frac{\pi n}{\alpha}}) + \\ + \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \theta_p e^{-\alpha (f_p^2 + b_m^2) \tau} \cos b_m (\beta - L_0) \left[ J_0(f_p r) + F_p Y_0(f_p r) \right]$$

## 7. Conclusions.

It is realized that the results obtained are not rigorous but it is felt that they constitute a fairly good approximation to the actual temperature distribution in the piston.

Further work might be done on the subject by solving an actual problem in order to check the practicability of

$$\text{right } g \text{ sin } \theta \Delta$$

Interfacing existing systems with new systems is a timely issue.

$$C = T_{\infty} + T_0 e^{-\left(\frac{2\pi}{\lambda}\right)^2 \left\{ -T_{\infty} + \left( \frac{2\pi}{\lambda} \right)^2 \left( \frac{1}{2} \ln \left( \frac{1}{1 - e^{-\left(\frac{2\pi}{\lambda}\right)^2}} \right) \right) \right\}}$$

$$(s_1, s_2) \text{ at } \left[ e^{\text{mod} \omega \frac{1}{\sqrt{2}}} + e^{\text{mod} \omega} \right]^{\frac{1}{2}} \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = 0.8 \underbrace{[3]}_{i=1} \underbrace{[3]}_{i=2} +$$

$$+ \left( \frac{1}{\frac{N\pi}{20}} \right) \{ -T \times \frac{N\pi}{20} \} \sin \left[ \frac{N\pi}{20} \right] e^{-j \omega T} \rightarrow 0T + jT = j$$

$$\left[ (s_1) \mathcal{A} \hat{q} + (s_2) \mathcal{A} \hat{v} \right] (s_3 - \ell) \rightarrow \text{and now } \stackrel{\text{if } (s_1 + s_2) \text{ is even}}{\rightarrow} \text{if } \begin{cases} s_1 \\ s_2 \end{cases} \begin{cases} \text{odd} \\ \text{even} \end{cases} +$$

### 3. Configuration.

Further work might be given on the subject of solvent selection to determine the best solvent for the polymerization of the polymer to be used in the production of the polymer.

the method used to determine  $P_p$ . By actual experiment on an engine the overall results of this work could be tested for accuracy and the percentage of error, if any could be found.

the meeting being held to determine Dr. Baileys' experience on  
an engine the results of this work could be tested  
for scientific and the bearings of this could be

found.

After this the Board of Engineers and Constructors of the  
U.S. Army Corps of Engineers, and the Bureau of Reclamation  
and the State of Colorado, and the City of Denver and the  
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## APPENDIX I

### GENERAL SOLUTION

To find a general solution to Fourier's Law of Heat Conduction for the case where the temperature should be analyzed in the unsteady state for a body best described in terms of cylindrical coordinates, we use:

$$\frac{\partial t}{\partial \tau} = \alpha \left( \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} \right) \quad (1)$$

where:

$$t = f(r, z, \tau)$$

Assume:  $t = \theta(\tau) \times R(r) \times Z(z)$

then:

$$\frac{1}{\alpha \theta} \frac{d\theta}{d\tau} = \frac{1}{R} \frac{d^2 R}{dr^2} + \frac{1}{rR} \frac{dR}{dr} + \frac{1}{Z} \frac{d^2 Z}{dz^2} \quad (2)$$

Since  $r$ ,  $z$ , and  $\tau$  are independent variables, we may say:

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{1}{rR} \frac{dR}{dr} = -\alpha^2 \quad (3)$$

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} = -b^2 \quad (4)$$

$$\frac{1}{\alpha \theta} \frac{d\theta}{d\tau} = -\alpha^2 - b^2 \quad (5)$$

Equation (3) may be rearranged as:

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \alpha^2 R = 0$$

which is Bessel's equation of the first kind of order zero.

## GENERAL SOLUTION

To find a general solution to Fourier's Law of Heat Conduction for the case where the temperature profile is subject to the unsteady state for a particular description in terms of cylindrical coordinates, we use:

$$(1) \quad \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} \right) \alpha = \frac{\partial \theta}{\partial t}$$

where:  $r = (x, y, z)$

$$(2) \quad S = \alpha K \times (r) \theta = f$$

then:

$$(3) \quad \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} = \frac{\partial \theta}{\partial t} - \frac{f}{\alpha K}$$

Since  $r$ ,  $\theta$ , and  $t$  are independent variables, we

see:

$$(4) \quad \theta = \frac{f}{\alpha K} \frac{1}{r} + \frac{1}{r^2} \frac{\partial \theta}{\partial r}$$

$$(5) \quad \theta = \frac{f}{\alpha K} \frac{1}{r}$$

$$(6) \quad \theta = \frac{f}{\alpha K} \frac{1}{r}$$

Equation (3) was then restated as:

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} = 0$$

which is Bessel's equation of the first kind of order zero.

Then:

$$R = A J_0(ar) + B Y_0(ar) \quad (6)$$

Equation (4) may be rearranged as:

$$\frac{d^2Z}{dz^2} + b^2 Z = 0$$

which yields:

$$Z = C \cos b_3 z + D \sin b_3 z \quad (7)$$

Equation (5) may be rearranged as:

$$\frac{d\theta}{dr} + \alpha \theta (a^2 + b^2) = 0$$

which yields:

$$\theta = E e^{-\alpha (a^2 + b^2)r} \quad (8)$$

But, since:  $t = \theta \times R \times Z$

we find using equations (6), (7), and (8), that:

$$t = E e^{-\alpha (a^2 + b^2)r} [C \cos b_3 z + D \sin b_3 z] [A J_0(ar) + B Y_0(ar)]$$

In this equation the constants  $a$ ,  $b$ ,  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  must be found so as to satisfy the boundary conditions of the specific problem involved.

$$(3) \quad (r_0) \mathcal{A} \otimes + (r_0) \mathcal{D} \mathcal{A} = \mathcal{R} \quad \text{Type: } \mathcal{D}$$

Equation (3) is a linear equation as:

$$0 = \mathcal{R} + P_1 \mathcal{A}$$

$$(4) \quad \text{if } \sin(\mathcal{A} + \mathcal{D} \mathcal{R}) = \mathcal{S} \quad \text{Type: } \mathcal{D}$$

Equation (4) is a linear equation as:

$$0 = (P_1 + P_2) \mathcal{R} + \frac{\mathcal{S}}{\mathcal{A}}$$

$$(5) \quad e^{\mathcal{A} + \mathcal{D} \mathcal{R}} = \mathcal{E} \quad \text{Type: } \mathcal{D}$$

$$\mathcal{E} \times \mathcal{R} \times \mathcal{A} = \mathcal{S} \quad \text{Type: } \mathcal{D}$$

From (5), (4) and (3), we find the definitions (6), (7), (8) and (9) are:

$$c = e^{\mathcal{A} + \mathcal{D} \mathcal{R}} \quad [C \sin(\mathcal{A} + \mathcal{D} \mathcal{R}) + \mathcal{R} \mathcal{S}]$$







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